Dimensional Analysis

Conversion factors – Converts one unit of measure to another

$\frac{\text{unit 1}}{\text{unit 2}}$ } factor

$$67.2 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = \quad \text{(Exact)}$$

$$67.2 \times 2.54 = 170.688 \text{ cm} \quad \text{(3)}$$

$$= 171 \text{ cm}$$

Process

- Parse the word problem
  - * making a drawing
    - finding the "desired"
    - finding the "given"
    - finding "equalities" (Conversion factors)
- Develop a roadmap
  - filling in known equalities
  - finding "bridging" equalities
- Writing the equation
- Checking the units
- Doing the math
- Sig figs

Given

A nearby lake holds 2.33×10^{15} L water. If a nearby city removes water at a rate of equality 3.34×10^{9} gallons of water/day, how many desired hours would the water last?

Equalities

4 quarts = 1 gallon, 1 quart = 0.94365 L,

24 hrs = 1 day, 60 min = 1 hr

Given unit = 2.33×10^{15} L water

Desired unit = ? hours

equalities 3.34×10^{9} gal = 1 day

Road Map

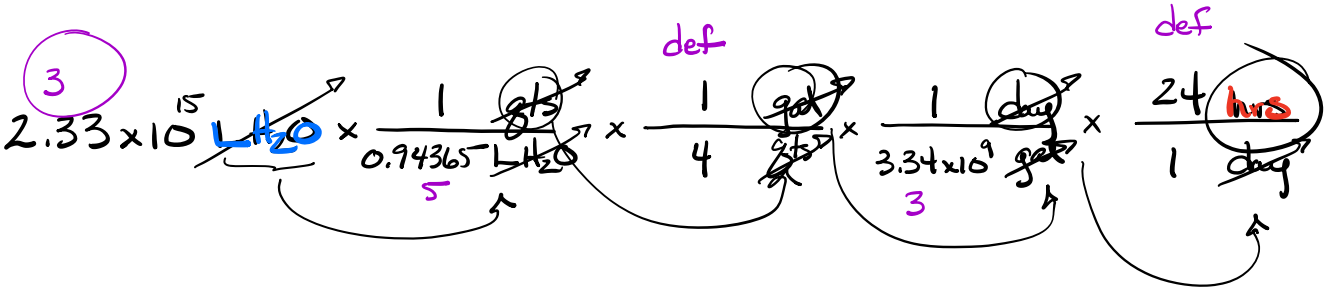
Given unit → Equalities linking units → Desired unit

L H_2O → ($\frac{0.94365\ L}{1\ qt}$) → qt → ($\frac{4\ qts}{1\ gal}$) → gal → ($\frac{3.34 \times 10^{9}\ gal}{1\ day}$) → day → ($\frac{1\ day}{24\ hrs}$) → hrs

Volume | Time

Bridging equality

Writing the equation



$= 2.33\ \underset{\text{EE}\ 10^x}{\text{E}}\ 15 \div 0.94365 \div 4 \div 3.34\ \underset{\text{EE}\ 10^x}{\text{E}}\ 9 \times 24$

$= 4435573.29785$ hrs

$= 4440000$ hrs or 4.44×10^6 hrs

$$\frac{A}{B} \times \frac{C}{1} = \frac{A \times C}{B \times 1} = \frac{A \times C}{B}$$

$$A \times \frac{1}{B} \times C$$

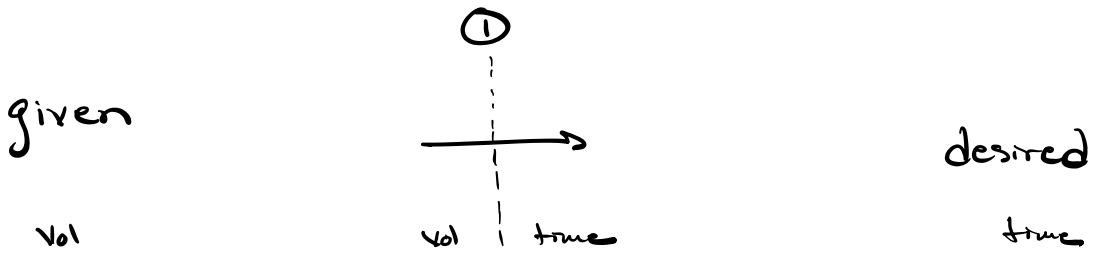
$$A \div B \times C$$

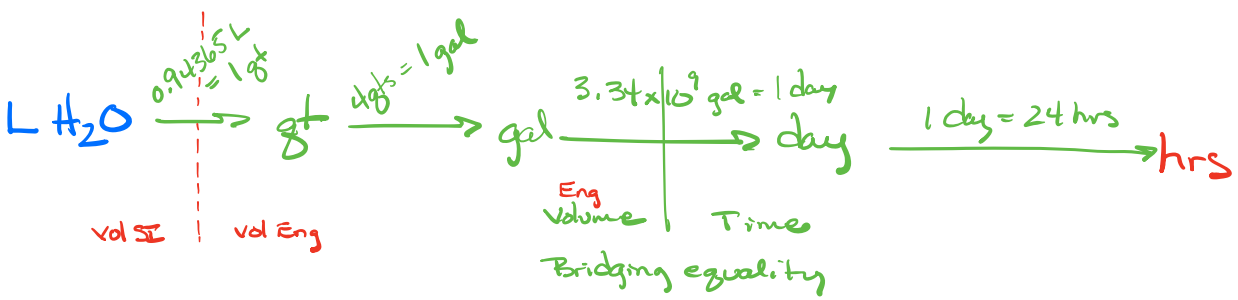
Road Map Keys

one way is sequential

given $\xrightarrow{①}$ A $\xrightarrow{②}$ B $\xrightarrow{③}$ C $\xrightarrow{④}$ desired

The more powerful skill is to find the bridge first.





3 Key Equalities to Memorize

	English		SI	
Length	1 in	=	2.54 cm	*Exact definition
Volume	1 gal	=	3.785 L	4 Sig figs
Mass	1 lbs	=	453.6 g	4 Sig figs

Problem # 3 of Dimensional Analysis Worksheet

The distance from Santa Cruz to Santa Barbara is 280 miles. If a car gets 23.6 miles per gallon, and the price of gas is 32 cents per liter, how much will it cost in gas to drive from Santa Cruz to Santa Barbara?

desired = dollars cost

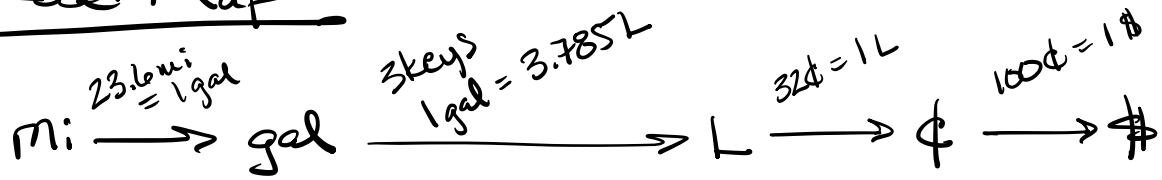
given = 280 miles

per = equality implies relationship

23.6 mi = 1 gallon

32 cents = 1 Liter

Road Map



2 3 4 measured 2 SF def

$$280\ \text{mi} \times \frac{1\ \text{gal}}{23.6\ \text{mi}} \times \frac{3.785\ \text{L}}{1\ \text{gal}} \times \frac{32\ ¢}{1\ \text{L}} \times \frac{1\ \$}{100\ ¢}$$

$$280 \div 23.6 \times 3.785 \times 32 \div 100 = \$14.370\,169\,4915$$

280. 3 SF

280 2 SF

$4.85 9/10 / gal

4.859

$14

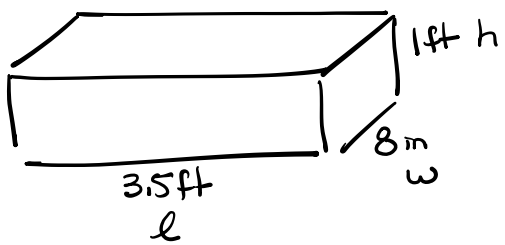
Converting Problems to images

Temperature Conversions

Percents as Conversions

Cubes & Squares

How many cubic meters of soil are needed to fill a flowerbox that is 3.5 feet long, 8 inches wide, and 1 foot deep?



$= \_\_\_ m^3$ Volume

Volume = $\ell \times w \times h$

ℓ 3.5ft $\xrightarrow{1ft = 12in}$ in $\xrightarrow[\text{Eng}]{1in = 2.54cm}$ cm $\xrightarrow[\text{SI}]{1cm = 1 \times 10^{-2} m}$ m

w 8in $\longrightarrow$ cm $\longrightarrow$ m

h 1ft $\longrightarrow$ in $\longrightarrow$ cm $\longrightarrow$ m

$\ell \times w \times h = m^3$

$$3.5\text{ft} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{2.54\text{cm}}{1\text{ in}} \times \frac{1\times10^{-2}\text{ m}}{1\text{ cm}} = 1.0668\text{ m}$$

(def, def, def; 2 sig figs; * def)

or

$$\times \frac{1\text{ m}}{100\text{ cm}} =$$

$$8\text{ in} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}} = 0.2032\text{ m}$$

(def, def; 1 sig fig)

$$1\text{ ft} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}} = 0.3048\text{ m}$$

(def, def, def; 1 sig fig)

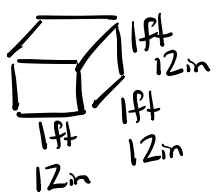
$$1.0668\text{ m} \times 0.2032\text{ m} \times 0.3048\text{ m} = 0.066072642\text{ m}^3$$

(2, 1, 1 sig figs)

$$= 0.07\text{ m}^3$$

$$3.5\text{ft} \times 1\text{ft} \times \left(8\text{in} \times \frac{1\text{ ft}}{12\text{ in}}\right) = 2.33\overline{3}\text{ ft}^3$$

$$\text{ft}^3 \rightarrow \text{in}^3 \rightarrow \text{cm}^3 \rightarrow \text{m}^3$$



$$1\text{ft}^3 = 12^3\text{ in}^3$$

$$1\times1\times1 = 1^3\text{ ft}^3 = 12^3\text{ in}^3$$

$$2.33\overline{3}\text{ ft}^3 \times \frac{12^3\text{ in}^3}{1\text{ ft}^3} \times \frac{2.54^3\text{ cm}^3}{1\text{ in}^3} \times \frac{1\text{ m}^3}{100^3\text{ cm}^3} = 0.066072642\text{ m}^3$$

$$= 0.07\text{ m}^3$$

°F	°C	K absolute Scale No negative values
212 °F	100 °C	373.15 K
180 div	100 div	100 div
32 °F	0 °C	273.15 K
negative values	−273.5 °C	0 Absolute Zero All motion Stops

°C → K Exact

°C + 273.15 = K

K → °C Exact

K − 273.15 = °C

°F → °C

def def

$$(°F - 32) \times \frac{100\,°C}{180\,°F} = °C$$

(°F − 32): phase shift; 100 °C / 180 °F: unit shift

°C → °F

$$°C \times \frac{180\,°F}{100\,°C} + 32\,°F = °F$$

$\frac{180\,°F}{100\,°C}$: unit shift; $+ 32\,°F$: phase shift

$$°F \longleftrightarrow °C \longleftrightarrow K$$

Constructing equalities from percents

Alcohol Content: 12.5% by volume

% = 1 part in 100

12.5 mL alcohol = 100 mL of wine

$$\frac{12.5\text{ mL Alcohol (Part)}}{100\text{ mL Wine (whole)}} \times 100 = 12.5\%$$

$$\text{Percent} = \frac{\text{Part}}{\text{whole}} \times 100 = \%$$

How percent is found

What is the percent by volume alcohol in a solution with a total volume of 537 mL and an alcohol volume of 23.6 mL?

$$\frac{\text{Part}}{\text{whole}} \times 100 = \frac{23.6\text{ mL alcohol}}{537\text{ mL Solution}} \times 100 = 4.394785\%$$

$$= \boxed{4.39\% \text{ by volume}}$$

How do we use Percent?

How many mL of alcohol would be in 780 mL of wine with a 4.39% by volume alcohol content?

$$4.39\% = \frac{4.39 \text{ mL alcohol}}{100 \text{ mL Wine}}$$

or

$$4.39 \text{ mL alcohol} = 100 \text{ mL Wine}$$

Road Map

mL wine ⟶ mL alcohol

$$780 \text{ mL Wine} \times \frac{4.39 \text{ mL alcohol}}{100 \text{ mL Wine}} = 34.242 \text{ mL alcohol}$$

2 sig figs ↑ 780 mL Wine

3 sig figs 4.39 mL alcohol

def 100 mL Wine

34.242 mL alcohol → 34 mL alcohol

$$780 \text{ gal wine} \times \frac{4.39 \text{ gal alcohol}}{100 \text{ gal wine}} = 34 \text{ gal alcohol}$$

% by volume $\frac{\text{mL part}}{100\ \text{mL whole}}$ or $\frac{\text{gal part}}{100\ \text{gal whole}}$

or $\frac{\text{L part}}{100\ \text{L whole}}$