Activity 1 - Math and the Calculator

Goals

- State the correct number of significant figures in a measurement.
- Round off a calculated answer to the correct number of significant figures.
- Give final answers with the correct number of significant figures.
- Write a number in scientific notation.

Pre-lab Questions *(answer these on a separate sheet using complete sentences)*

1. How does a measured number differ from an exact number?
2. Why does a measurement have a limited number of significant figures?
3. How do you determine the number of significant figures for a calculated answer?
4. What are the rules for rounding off numbers?

Concepts to Review

Significant figures
Exact numbers
Mathematical operations with significant figures
Scientific notation
Calculator use

Introduction

In the sciences, matter is characterized using a wide variety of numbers. Measured numbers are different from counted or defined numbers. Any budding chemist must learn how to distinguish measurements from exact numbers. Measurements may be used in calculations and the calculated numbers must accurately reflect the quality of the measured number used. The following information and exercises will help prepare you for the mathematics involved in learning chemistry.

Required Materials

Scientific calculator

A. Measured and Exact Numbers

Suppose you used a bathroom scale this morning to determine that your weight is 145 lb. The scale is a measuring device and the number in 145 lb is called a *measured number*. When numbers are obtained by counting objects or using a definition, they are called *exact numbers*. Suppose you counted 22 people in your lab. The number 22 is an exact number because you did not need to use a measuring device. The relationships between units within the metric system (S.I.) or within the American system are defined numbers, which makes them exact numbers as well. For example, the numbers in definitions such as 100 cm in 1 meter and 12 inches in 1 foot are exact. See Example 1.

Example 1.

Describe each of the following as a measured or exact number.

a. 14 inches
b. 14 pencils
c. 60 minutes in 1 hour
d. 7.5 kg

Solution:

a. measured
b. exact (counted number)
c. exact (definition)
d. measured

B. Scientific Notation

In scientific work, small numbers such as 0.000000025 m and large numbers such as 4,000,000 g are often expressed using powers of 10; the above numbers are 2.5×10^{-8} m and 4×10^{6} g. The values 2.5 and 4 are *coefficients*; the values 10^{-8} and 10^{6} are powers of ten (See Table 1 and Example 2). The rules for converting standard numbers to scientific notation are given below.

Table 1. Examples of exponential notation

Standard number		Power of ten	Standard number		Power of ten
10,000	=	10^{4}	0.1	=	10^{-1}
1000	=	10^{3}	0.01	=	10^{-2}
100	=	10^{2}	0.001	=	10^{-3}
10	=	10^{1}	0.0001	=	10^{-4}

Rules for Writing Numbers in Scientific Notation

For numbers larger than 10:

a. Move the decimal point to the left until it follows the first digit in the number.
b. Write a power of ten that is equal to the number of places the decimal point was moved to the left.

For numbers smaller than 1:

a. Move the decimal point to the right until it is located after the first digit in the number.
b. Write a negative power of ten that is equal to the number of places the decimal point was moved to the right.

Example 2.

Write the following standard numbers in scientific notation:

a. 35,000 b. 608 c. 0.0000815

Solution:

				coefficient		power of ten
a.	35,000	= 3 5 0 0 0.	=	3.5	×	10^4
b.	608	= 6 0 8.	=	6.08	×	10^2
c.	0.0000815	= 0.0 0 0 0 8 1 5	=	8.15	×	10^{-5}

C. Significant Figures

In measured numbers, all the reported figures are called *significant figures*. The first significant figure is the first nonzero digit. The last significant figure is always the estimated digit. Zeros between other digits or at the end of the decimal part of a number (i.e., trailing zeros in a number with a decimal part) are counted as significant figures. Leading zeros after the decimal point are *not significant;* they are placeholders. Trailing zeros are not *significant* in numbers equal to or greater than 10 if there is no decimal point present; they are placeholders needed to express the magnitude of the number.

In a number written in scientific notation, *all* the figures in the coefficient *are significant*. Examples of counting significant figures in measured numbers are given in Table 3.2 and Example 3.3.

Table 2. Examples of Counting Significant Figures.

Measurement	Significant Figures	Reason
455.2 cm	4	All nonzero digits are significant
0.800 m	3	The trailing zeros in the decimal part are significant
50.2 L	3	A zero between nonzero digits is significant
0.0005 lb	1	Leading zeros are not significant
25,000 ft	2	Placeholder zeros are not significant
3.20×10^4 g	3	All the digits in a coefficient are significant

Example 3.

State the number of significant figures in each of the following measured numbers:

a. 0.00580 m b. 132.08 g c. 1.5×10^3 mL

Solution:

a. Three significant figures. The zeros immediately after the decimal point are placeholder zeros; the last zero is a trailing zero in the decimal part.

b. Five significant figures. The zero between nonzero digits is significant.

c. Two significant figures. All the digits in the *coefficient* of a number in scientific notation are significant.

D. Rounding Off

Often, you will use a measurement in a mathematical operation such as multiplication, division, addition or subtraction. When the calculator display shows more numbers than the measurements support, it is necessary to *round off* the calculated answer. If the numbers to be dropped begin with a number **less than 5,** they are simply dropped. However, if the numbers dropped begin with **a number greater than 5,** the value of the last *retained digit* is increased by 1. If the number to be rounded is **exactly 5** (all figures following 5, if any, are 0)**,** round an odd number to the next highest even number and leave an even number unrounded. For a calculator display already in scientific notation, round off the coefficient to report the correct number of significant figures (sig figs).

On your calculator, an answer may appear in scientific notation, which means that a coefficient and a power of ten are shown. (See Table 3 and Example 4.) In scientific notation, the correct number of sig figs is shown in the coefficient. Since calculators know nothing of sig figs, the numbers shown in a calculator display before the power of ten must be rounded to the correct number of sig figs. Be *sure* to keep the power of ten when recording your final answers!

Please be advised that Table 3 may not represent the exact output of your calculator. Now is the time to make yourself familiar with your calculator, especially regarding how to perform mathematical operations with exponential numbers. *See your instructor before your first exam if you are not successful in mastering this task.*

The exponential function key is usually represented by an "EE" or "EXP" key and acts as "× 10". Do not use the exponent key **and then** multiply by 10. Only the exponent key is necessary. The consequence of making this mistake is that your results will be consistently too high by a power of ten.

Table 3. Examples of Writing Calculator Results in Scientific Notation.

Calculator display	Number of significant figures to be shown in coefficient	Rounded and written in scientific notation
2.512^{05} or 2.152E5	2	2.5×10^{5}
4.1585^{12} or 4.1585E12	3	4.16×10^{12}
8.775^{-08} or 8.775E-8	3	8.78×10^{-8}

Example 4

Round off each of the following calculator displays to report answers with three significant figures as well as two significant figures.

a. 75.6243 b. 0.528392 c. 387,600 d. 8.027^{-04} (displayed on some as 8.027E-4)

Solution:

	Three significant figures	Two significant figures
75.6243	75.6	76
0.528392	0.528	0.53
387,600	388,000	390,000
8.027^{-04}	8.03×10^{-4}	8.0×10^{-4}

E. Multiplication and Division of Measured Numbers

If a calculated answer is obtained from multiplication and/or division, it is rounded off to the same number of significant figures as the measured number with the *fewest* significant figures. See Examples 5 and 6.

Example 5

Solve: $\frac{(0.025\text{ m})(4.62\text{ g})}{3.44\text{ s}} =$

Solution: On the calculator, the steps are:

Enter keys	Display reads	
0.025	0.025	two significant figures
×	0.025	
4.62	4.62	three significant figures
=	0.1155	
	0.1155	("Enter keys" left blank since no division sign on computer keyboard)
3.44	3.44	three significant figures
=	0.033575581	Calculator display to 9 decimal places
	$0.034\frac{\text{m}\times\text{g}}{\text{s}}$	final answer rounded to **two** significant figures with the correct units.

Example 6

Solve: $\frac{3.4\times10^{-4}\text{g}}{2.75\times10^{8}\text{mL}} =$

Solution: On the calculator, the steps are:

Enter keys	Display reads	
3.4	3.4	two significant figures
EXP (or EE)	3.4^{00} or 3.4E	
4	3.4^{04} or 3.4E4	
(-) or (±)	3.4^{-04} or 3.4E-4	
	3.4^{-04} or 3.4E-4	"Enter keys" left blank as above
2.75	2.75	
EXP (or EE)	2.75^{00} or 2.75E	
8	2.75^{08} or 2.75E8	
=	1.2363636^{-12} or 1.2363636E-12	calculator display to 7 decimal places plus two digit exponent
	$1.2\times10^{-12}\frac{\text{g}}{\text{mL}}$	Coefficient in **final answer** rounded to *two* significant figures; units must be included in your written answer.

F. Addition and Subtraction of Measured Numbers

After you have added or subtracted measured numbers, you may need to round off the result. An answer from addition or subtraction has the last significant figure in the column where **all** of the numbers added or subtracted **also** have significant figures. See Examples 7, 8 and 9.

Example 7

Addition: 42.11 cm + 4.056 cm + 30.1 cm =

Solution:

```
    42.11
+    4.056
+   30.1      digits in 30.1 end at the tenth's place; all other numbers go
further than this.
    76.266    calculator display

    76.3 cm   correct answer rounded to give digit in tenth's place
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accompanied by proper units.

Example 8

Subtraction: 14.621 g - 3.39 g =

Solution:

```
    14.621
-    3.39     digits in 3.39 end at the hundredth's place; all others go further.
    11.231    calculator display

    11.23 g   correct answer rounded to give final significant digit in the hundredth's place,
              with units included.
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Example 9

Addition: 1200 m + 14 m + 1.11 m =

Solution:

```
  1200        digits in 1200 end at the hundred's place; all others go further
+   14
+    1.11
  1215.11     calculator display

  1200 m      correct answer rounded to give the last significant digit in the hundred's
              place, with units included.
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Activity 1 - Math and the Calculator Worksheet

Name_____

Section_____ **Date**_____

Exercise A. Measured and Exact Numbers

Circle "M" or "E" to indicate whether each of the following numbers is measured or exact:

5 books	M E	12 roses	M E
5 lb	M E	12 inches in 1 foot	M E
9.25 g	M E	361 miles	M E
0.035 kg	M E	100 cm in 1 m	M E

Exercise B. Scientific Notation

Write the following numbers in scientific notation:

4,450,000	_____	0.00032	_____
38,000	_____	25.2	_____
0.0000000021	_____	0.0505	_____

Write the following as standard numbers:

4×10^{2}	_____	3×10^{-4}	_____
5×10^{3}	_____	8.2×10^{-3}	_____
3.15×10^{5}	_____	2.46×10^{-6}	_____

Exercise C. Significant Figures

State the number of significant figures in each of the following measured quantities:

4.5 m	_____	204.52 g	_____
0.0004 L	_____	625,000 mm	_____
850 lb	_____	34.80 km	_____
2.50×10^{-3} L	_____	8×10^{5} g	_____

Exercise D. Rounding Off

Round off each of the following to the number of significant figures indicated. Don't forget placeholder zeros when necessary!

	Three significant Figures	Two Significant Figures
e.g.: 0.4108 g	0.411 g	0.41 g
143.63212 mi		
532,800 ft		
5.448×10^2 yrs		
0.00858345 mm		

Exercise E. Multiplication and Division

Do the following multiplication and division calculations. **Give a final answer with the correct number of significant figures and the correct units. Units can cancel or multiply just like number factors**.

e.g.: 4.5 ergs × 0.281 in = 1.3 erg·in

0.1184 cm × 8.00 cm × 0.0345 cm = _____

$(2.5 \times 10^4 \text{ m/s}) \times (5.0 \times 10^{-7} \text{ s}) =$ _____

$$\frac{(42.2 \text{ L})(1.45 \text{ atm})}{(4.8 \text{ mol})(0.0821 \frac{\text{L atm}}{\text{mol K}})} =$$ _____

Exercise F. Addition and Subtraction

Do the following addition and subtraction calculations. Give a final answer with the correct number of significant figures and units.

13.45 mL + 0.4552 mL = _____

145.5 m + 86.58 m + 1045 m = _____

1315 mi + 200 mi + 1100 mi = _____

245.625 g - 80.2 g = _____

4.62 cm - 0.885 cm = _____

Questions and Problems

1. How can you distinguish an exact number from a measured number?

2. In the scientific community, the last digit in a measured number that is still significant is sometimes called the estimated digit. Circle or underline the estimated digit in each of the following measurements:

   1.5 cm 4500 mi. 0.0782 m 42.50 g

3. Bill and Beverly have measured the sides of a rectangle. Each recorded the length as 6.7 cm and the width as 3.9 cm. When Bill calculates the area, he gives an answer of 26.13 cm^2. However, Beverly gives her answer for the area as 26 cm^2.

   a. Give the most likely explanation for the difference between the two calculated answers despite the fact that both students used the same measurements.

   b. You are going to tutor Bill. Using complete sentences, describe how you would help him to understand why his answer is wrong and Beverly's is right.

Measurements in Your Daily Life

4. Throughout a typical day, list at least eight numbers (and units) you might use such as measurements, prices, definitions, cooking quantities, gasoline purchases, prescription dosages, etc. Identify each number as exact or measured. Explain your choice. (Did you use a measuring tool, or did you count out something, or use a definition?)

Number used	Type of Number	Explanation
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

5. In the list above, were the numbers you used mostly measured numbers, or mostly exact numbers?

6. List the names and abbreviations of five units (metric or American) of the measurement you used. Give the property measured (weight, mass, volume, distance, etc.)

Unit of Measurement	Abbreviation	Property Measured
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____